SEMICLASSICAL QUANTIZATION OF $SU(3)$ SKYRMIONS [1]

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ABSTRACT

Semiclassical quantization of the SU(3)-skyrmion zero modes is performed by means of the collective coordinate method. The quantization condition known for SU(2) solitons quantized with SU(3) collective coordinates is generalized for SU(3) skyrmions with strangeness content different from zero. Quantization of the dipole-type configuration with large strangeness content found recently is considered as an example and the spectrum and the mass splittings of the quantized states are estimated. The energy and baryon number density of SU(3) skyrmions are presented in the form emphasizing their symmetry in different SU(2) subgroups of SU(3), and the lower boundary for the static energy of SU(3) skyrmions is derived.

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1 Introduction

The chiral soliton approach proposed at first by Skyrme [1] allows one to describe the properties of baryons with fairly good accuracy [2]-[4]. Considerable progress has been made recently also in understanding the properties of few-nucleon systems [5]-[7]. Moreover, this approach allows some predictions for the spectrum of states with baryon number $B > 1$ [8]-[13]. The quantization of the bound states of skyrmions, primarily their zero modes, is a necessary step towards realization of this approach. Different aspects of this problem have been considered, beginning with the papers [2], [13] and [8, 9, 14]; however, more general treatment allowing the consideration of arbitrary $SU(3)$ skyrmions was lacking until recently.

In the sector with $B = 2$ besides the $SO(3)$ hedgehog with the lowest quantum state interpreted as an H-dibaryon [8, 9], the $SU(2)$ torus - a bound $B = 2$ state - was discovered 10 years ago [15]. In the flavor-symmetric (FS) case, when all meson masses in the Lagrangian are equal to the pion mass, there are three degenerate tori in the (u, d), (d, s) and (u, s) $SU(2)$ subgroups of $SU(3)$. In the flavor symmetry broken (FSB) case the (u, s) and (d, s) tori are degenerate and heavier than the (u, d) torus. Another local minimum with large strangeness content was found recently in the $SU(3)$ extension of the model [16]. This configuration is of molecular type and consists of two interacting $B = 1$ skyrmions located in different $SU(2)$ subgroups of $SU(3)$, (u, s) and (d, s). The attraction between two $B = 1$ skyrmions in optimal orientation which led to the formation of the torus-like state is not sufficient for this when both skyrmions are located in different $SU(2)$ subgroups of $SU(3)$ and interact due only to one common degree of freedom. To find this configuration a special algorithm was developed allowing for the minimization of an energy functional depending on eight functions of three variables [16]. The position of the known $B = 2$ classical configurations representing local minima in $SU(3)$ configuration space is shown on Fig.1 in the plane with the scalar strangeness content C_S [17] as Y axis and the difference of the U- and D-contents as X-axis. Since the sum of all scalar contents is equal to unity, they are defined uniquely at

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each point of this plot. The $SO(3)$ hedgehog (1) has all contents equal to $1/3$ [12]. Intuitively this is clear, since the basis for the $SO(3)$ solitons is formed by the matrices $\lambda_2, -\lambda_5, \lambda_7$ and they are located in three $SU(2)$ subgroups of $SU(3)$ on equal footing. The three tori in three different $SU(2)$ subgroups of $SU(3)$ are denoted by the labels (2), (3) and (4), the $u-d$ symmetric state (2) with $C_S = 0$ being of special interest. The configurations (3) and (4) can be connected by isorotation in the (u, d) subgroup. The dipole type state (5) found recently [16] has a binding energy about half of that of the torus, i.e., about 0.04 of the mass of the $B = 1$ skyrmion.

The zero modes of solitons have been quantized previously in a few cases: for $SU(2)$ solitons rotated in the $SU(2)$ [2] as well as in the $SU(3)$ configuration space of collective coordinates [13, 8, 14], and also for $SO(3)$ solitons [8, 9]. In the case of $SU(2)$ solitons rotated in $SU(3)$ space the quantization condition known as the Guadagnini condition [13] was established; see also [18].

The quantization of the $SU(2)$ $B = 1$ hedgehog yields the spectrum of baryons, mainly the octet and decuplet, and moderate agreement with the data has been achieved [4]. Quantization of the $SU(2)$ torus in the $SU(3)$ space of collective coordinates leads to predictions of a rich spectrum of strange dibaryons [19, 11]. Most of them are probably unbound if a natural assumption concerning the poorly known Casimir energy of the torus-like solitons is made; see also the discussion in the last section.

However, these solitons are only particular cases, since other types of solitons exist, e.g. the above-mentioned solitons of dipole type with large strangeness content [16], point (5) on Fig.1. In general, one should expect that the map of the local minima in the $SU(3)$ configuration space will come more and more complicated with as the baryon number of the configuration increases. In some cases the local minima corresponding to larger strangeness content may have greater binding energy than configurations with small or zero C_S. Therefore, a quantization procedure for arbitrary $SU(3)$ solitons should be developed. This is a subject of the present paper ([20] contains a preliminary short version).

2 The Wess-Zumino-Witten term

Let us consider the Wess-Zumino (WZ) term in the action which defines the quantum numbers of the system in the quantization procedure. It was written by E.Witten in the elegant form [21]:

$$S^{WZ} = \frac{-iN_c}{240\pi^2}\epsilon_{\mu\nu\alpha\beta\gamma}\int_\Omega Tr\bar{L}_\mu\bar{L}_\nu\bar{L}_\alpha\bar{L}_\beta\bar{L}_\gamma d^5x', \tag{1}$$

where Ω is the 5-dimensional region with 4-dimensional space-time as its boundary, N_C is the number of colors of the underlying QCD, and $\bar{L}_\mu = U^\dagger d_\mu U$. As usual, we introduce time-dependent collective coordinates for the quantization of zero modes according to the relation: $U(\vec{r}, t) = A(t)U_0(\vec{r})A^\dagger(t)$. Integration by parts is possible then in the expression for the WZ-term in the action, and we obtain for the WZ-term contribution to the Lagrangian of the system:

$$L^{WZ} = \frac{-iN_c}{48\pi^2}\epsilon_{\alpha\beta\gamma}\int TrA^\dagger\dot{A}(R_\alpha R_\beta R_\gamma + L_\alpha L_\beta L_\gamma)d^3x, \tag{2}$$

where $L_\alpha = U_0^\dagger d_\alpha U_0 = iL_{k,\alpha}\lambda_k$ and $R_\alpha = d_\alpha U_0 U_0^\dagger = U_0 L_\alpha U_0^\dagger$, or

$$L^{WZ} = \frac{N_c}{24\pi^2}\int\sum_{k=1}^{k=8}\omega_k WZ_k d^3x = \sum_{k=1}^{k=8}\omega_k L_k^{WZ}, \tag{3}$$

with the angular velocities of rotation in the configuration space defined in the usual way, $A^\dagger \dot{A} = -\frac{i}{2}\omega_k\lambda_k$. Summation over repeated indices is assumed here and below. The functions WZ_k can be expressed through the chiral derivatives $\vec{L}_k$:

$$WZ_i = WZ_i^R + WZ_i^L = (R_{ik}(U_0) + \delta_{ik})WZ_k^L, \qquad (4a)$$

$i, k = 1, ...8$, and

$$\begin{aligned}
WZ_1^L &= -(L_1, L_4L_5 + L_6L_7) - (L_2L_3L_8)/\sqrt{3} - 2(L_8, L_4L_7 - L_5L_6)/\sqrt{3} \\
WZ_2^L &= -(L_2, L_4L_5 + L_6L_7) - (L_3L_1L_8)/\sqrt{3} - 2(L_8, L_4L_6 + L_5L_7)/\sqrt{3} \\
WZ_3^L &= -(L_3, L_4L_5 + L_6L_7) - (L_1L_2L_8)/\sqrt{3} - 2(L_8, L_4L_5 - L_6L_7)/\sqrt{3} \\
WZ_4^L &= -(L_4, L_1L_2 - L_6L_7) - (L_3L_5L_8)/\sqrt{3} + 2(\tilde{L}_8, L_1L_7 + L_2L_6)/\sqrt{3} \\
WZ_5^L &= -(L_5, L_1L_2 - L_6L_7) + (L_3L_4L_8)/\sqrt{3} - 2(\tilde{L}_8, L_1L_6 - L_2L_7)/\sqrt{3} \\
WZ_6^L &= (L_6, L_1L_2 + L_4L_5) + (L_3L_7L_8)/\sqrt{3} - 2(\tilde{\tilde{L}}_8, L_1L_5 - L_2L_4)/\sqrt{3} \\
WZ_7^L &= (L_7, L_1L_2 + L_4L_5) - (L_3L_6L_8)/\sqrt{3} + 2(\tilde{\tilde{L}}_8, L_1L_4 + L_2L_5)/\sqrt{3} \\
WZ_8^L &= -\sqrt{3}(L_1L_2L_3) + (L_8L_4L_5) + (L_8L_6L_7),
\end{aligned} \qquad (5)$$

where $(L_1L_2L_3)$ denotes the mixed product of vectors $\vec{L}_1$, $\vec{L}_2$, $\vec{L}_3$, etc. and $\tilde{L}_3 = (L_3 + \sqrt{3}L_8)/2$, $\tilde{L}_8 = (\sqrt{3}L_3 - L_8)/2$, $\tilde{\tilde{L}}_3 = (-L_3 + \sqrt{3}L_8)/2$, $\tilde{\tilde{L}}_8 = (\sqrt{3}L_3 + L_8)/2$ are the third and eighth components of the chiral derivatives in the (u, s) and (d, s) $SU(2)$-subgroups. Here $[\tilde{\vec{L}}_3\tilde{\tilde{\vec{L}}}_8] = -[\tilde{\vec{L}}_3\tilde{\vec{L}}_8]$, etc. $R_{ik}(U_0) = \frac{1}{2}Tr\lambda_i U_0 \lambda_k U_0^\dagger$ is a real orthogonal matrix, and WZ_i^R are defined by the expressions (5) with the substitution $\vec{L}_k \to \vec{R}_k$. Relations similar to (5) can be obtained for $\widetilde{WZ}_3$ and $\widetilde{WZ}_8$; they are analogs of WZ_3 and WZ_8 for the (u, s) or (d, s) $SU(2)$ subgroups, thus clarifying the symmetry of the WZ-term in the different $SU(2)$ subgroups of $SU(3)$.

The baryon number of the $SU(3)$ skyrmions can be written also in terms of $\vec{L}_i$ in a form where its symmetry in the different $SU(2)$ subgroups of $SU(3)$ is obvious:

$$B = -\frac{1}{2\pi^2}\int\left((\vec{L}_1\vec{L}_2\vec{L}_3) + (\vec{L}_4\vec{L}_5\tilde{\vec{L}}_3) + (\vec{L}_6\vec{L}_7\tilde{\tilde{\vec{L}}}_3) + \frac{1}{2}[(\vec{L}_1, \vec{L}_4\vec{L}_7 - \vec{L}_5\vec{L}_6) + (\vec{L}_2, \vec{L}_4\vec{L}_6 + \vec{L}_5\vec{L}_7)]\right)d^3r. \qquad (6)$$

The contributions of the three $SU(2)$ subgroups enter the baryon number on equal footing. In addition, mixed terms corresponding to the interaction of the chiral fields from different subgroups are present also.

It should be noted that the results of calculating the WZ-term according to (5) depend on the orientation of the soliton in the $SU(3)$ configuration space. When solitons are located in the (u, d) $SU(2)$ subgroup of $SU(3)$ only L_1, L_2 and L_3 are different from zero, WZ_8^R and WZ_8^L are both proportional to the B-number density, and the well known quantization condition by Guadagnini [13] rederived in [18],

$$Y_R = \frac{2}{\sqrt{3}}dL^{WZ}/d\omega_8 = N_cB/3, \qquad (7)$$

applies, where Y_R is the so-called right hypercharge characterizing the $SU(3)$ irrep under consideration. This relation is generalized to [20]

$$Y_R^{min} = \frac{2}{\sqrt{3}}dL^{WZ}/d\omega_8 = N_cB(1 - 3C_S)/3, \qquad (8)$$

where the scalar strangeness content C_S is defined in terms of the real parts of the diagonal matrix elements of the matrix U:

$$C_S = \frac{< 1 - ReU_{33} >}{< 3 - Re(U_{11} + U_{22} + U_{33}) >}, \tag{9}$$

and $<>$ means averaging or integration over the whole 3-dimensional space [17]. This formula was checked in several cases.

a) One can rotate any $SU(2)$ soliton of the (u,d) subgroup by an arbitrary constant $SU(3)$ matrix containing $U_4 = exp(-i\nu\lambda_4)$. In this case $C_S = \frac{1}{2}sin^2\nu$ [17], and both WZ_8^R, WZ_8^L are proportional to $R_{88} = 1 - \frac{3}{2}sin^2\nu$. As a result, the relation (8) is fulfilled exactly. Solitons (3) and (4) on Fig.1 can be obtained from the (u,d) soliton (2) by means of U_4 or U_2U_4 rotations and satisfy relation (8). For example, when the skyrmion is located in the (u,s) $SU(2)$ subgroup of $SU(3)$ we have

$$L^{WZ}(u,s) = -\frac{\sqrt{3}N_C B}{12}(\omega_8 - \sqrt{3}\omega_3). \tag{10a}$$

For skyrmions in the (d,s) $SU(2)$ subgroup

$$L^{WZ}(d,s) = -\frac{\sqrt{3}N_C B}{12}(\omega_8 + \sqrt{3}\omega_3). \tag{10b}$$

Since $C_S = 0.5$ in both cases [17], relation (8) holds. To derive $(10a,b)$ we have noted that if the soliton is located in any $SU(2)$ subgroup of $SU(3)$ two terms in (2) and $(4a)$ give equal contributions.

b) For the $SO(3)$ hedgehog the strangeness content was calculated previously, $C_S = 1/3$, [12, 11] and $L^{WZ} = 0$ according to [8], at least for periodic $A(t)$ [9]. The standard assumption that the angular velocities are constant corresponds to (quasi)periodic behaviour of $A(t)$, so, relation (8) is satisfied.

c) We obtained the relation (8) numerically for solitons of the form [16]

$$U = U_L(u,s)U(u,d)U_R(d,s), \tag{11}$$

with $U(u,d) = \exp(ia\lambda_2)\exp(ib\lambda_3)$ and $U_L(u,s)$ and $U_R(d,s)$ being deformed interacting $B = 1$ $SU(2)$ hedgehogs. For this ansatz we had for the rotated $SU(3)$ Cartan- Maurer currents [16]:

$$\begin{aligned}
&L^r_{1i} = s_a c_a l_{3i}, && L^r_{2i} = d_i a, \\
&L^r_{3i} = (c_{2a} l_{3i} - r_{3i})/2 + d_i b, && L^r_{4i} = c_a l_{1i}, \\
&L^r_{5i} = c_a l_{2i}, && L^r_{6i} = s_a l_{1i} + r_{1i}(b), \\
&L^r_{7i} = s_a l_{2i} + r_{2i}(b), && L^r_{8i} = \sqrt{3}(l_{3i} + r_{3i})/2.
\end{aligned} \tag{12}$$

in terms of the $SU(2)$ C-M currents $l_{k,i}$ and $r_{k,i}$ $(i,k = 1,2,3)$ and the functions a and b, with $r_1(b) = c_b r_1 - s_b r_2$, $r_2(b) = c_b r_2 + s_b r_1$, $c_b = cosb$, $s_b = sinb$, $c_a = cosa$, etc.
$i\vec{l}_k\bar{\tau}_k = U_L^\dagger \vec{d}U_L$, $i\vec{r}_k\tilde{\tau}_k = \vec{d}U_R U_R^\dagger$, $k = 1,2,3$. Here $\bar{U}_L(u,s) = f_0 + i\bar{\tau}_k f_k$, $\tilde{U}_R(d,s) = q_0 + i\tilde{\tau}_k q_k$, $k = 1,2,3$, $\bar{\tau}$ and $\tilde{\tau}$ are the Pauli matrices corresponding to the (u,s) and (d,s) $SU(2)$ subgroups, and $f_0^2 + ... + f_3^2 = 1$, $q_0^2 + ... + q_3^2 = 1$.

$L_i^r = TL_iT^\dagger$, $U_0 = VT$, $V = U(u,s)exp(ia\lambda_2)$, $T = exp(ib\lambda_3)U(d,s)$. The chirally invariant quantities, B-number density (6), and the second- order and Skyrme term contributions to the static energy have the same form in terms of L_{ki} and L^r_{ki}. The formula $(4a)$ should be written then as

$$WZ_i = [R_{ik}(V) + R_{ik}(T^\dagger)]WZ_k^L, \tag{4b}$$

with WZ_k^L given in terms of L_n^r according to (5). In the following we shall omit the index "r" everywhere. Relations $(10a, b)$ can be checked easily with the help of $(4), (5)$ and (12).

Using (12) and (5) we obtain

$$WZ_8^L = \frac{\sqrt{3}}{2}((\vec{l}_1\vec{l}_2\vec{l}_3) + (\vec{r}_1\vec{r}_2\vec{r}_3) + s_a[(\vec{l}_1\vec{r}_2 - \vec{r}_1\vec{l}_2, \vec{l}_3 + \vec{r}_3) - (\vec{d}s_a\vec{l}_3, \vec{r}_3 - 2\vec{d}b)]) \qquad (13)$$

It follows from (13) and (4) that at large relative distances, for arbitrary but not overlapping solitons, and for $a = 0$, we have

$$Y_R^{min} = \frac{2}{\sqrt{3}}L_8^{WZ} = \frac{1}{2\sqrt{3}\pi^2}\int WZ_8^L d^3x = \frac{1}{4\pi^2}\int[(\vec{l}_1\vec{l}_2\vec{l}_3) + (\vec{r}_1\vec{r}_2\vec{r}_3)]d^3x = -(B_L + B_R)/2, \qquad (14)$$

where B_L and B_R are the baryon numbers located in the left (u, s) and right (d, s) $SU(2)$ subgroups of $SU(3)$. Relation (8) holds since $C_S = 1/2$ for both (u, s) and (d, s) skyrmions. Equation (14) does not hold in the general case for overlapping solitons, since there is no conservation law for the components of the Wess-Zumino term.

For the strange skyrmion molecule [16] we should calculate $(3), (5), (8)$ with $WZ_8 = (R_{8k}(V) + R_{k8}(T))WZ_k^L$. The contribution $-(B_L + B_R)/2$ also appears with some additional terms which turn out to be small numerically. We obtained $C_S = 0.475$ and $Y_R^{min} = -0.87$ in the FSB case, so relation (8) is satisfied with good accuracy.

It is natural to assume that (8) is valid with good accuracy for any $SU(3)$ skyrmions. However, corrections to this relation are not excluded by our treatment.

3 Rotation and static energy

We start with the well known Lagrangian of the Skyrme model widely used in the literature since [2]. It depends on the parameters $F_\pi = 186 Mev$ (experimental value) and the Skyrme parameter e:

$$L_{Sk} = -\frac{F_\pi^2}{16}Tr\bar{L}_\mu\bar{L}^\mu + \frac{1}{32e^2}Tr(\bar{L}_\mu\bar{L}_\nu - \bar{L}_\nu\bar{L}_\mu)^2 + L_M \qquad (15)$$

We take $e = 4.12$, close to the value suitable for describing, with a bit more complicated Lagrangian, the mass splittings inside the $SU(3)$ multiplets of baryons [4]. The chiral and flavor-symmetry-breaking mass terms L_M in (15) depending on meson masses will be described in detail in Section 4.

The expression for the rotation energy density of the system depending on the angular velocities of rotations in the $SU(3)$ collective coordinate space defined in Section 2 can be written in more compact form than previously [20, 16]:

$$L_{rot} = \frac{F_\pi^2}{32}(\tilde{\omega}_1^2 + \tilde{\omega}_2^2 + ... + \tilde{\omega}_8^2) +$$

$$+\frac{1}{16e^2}\Bigg\{(\vec{s}_{12} + \vec{s}_{45})^2 + (\vec{s}_{45} + \vec{s}_{67})^2 + (\vec{s}_{67} - \vec{s}_{12})^2 + \frac{1}{2}\Big((2\vec{s}_{13} - \vec{s}_{46} - \vec{s}_{57})^2 + (2\vec{s}_{23} + \vec{s}_{47} - \vec{s}_{56})^2 +$$

$$+(2\tilde{\vec{s}}_{34} + \vec{s}_{16} - \vec{s}_{27})^2 + (2\tilde{\vec{s}}_{35} + \vec{s}_{17} + \vec{s}_{26})^2 + (2\tilde{\vec{s}}_{36} + \vec{s}_{14} + \vec{s}_{25})^2 + (2\tilde{\vec{s}}_{37} + \vec{s}_{15} - \vec{s}_{24})^2\Big)\Bigg\} \qquad (16)$$

Here $\vec{s}_{ik} = \tilde{\omega}_i\vec{L}_k - \tilde{\omega}_k\vec{L}_i$, $i, k = 1, 2...8$ are the $SU(3)$ indices, and $\tilde{\vec{s}}_{34} = (\vec{s}_{34} + \sqrt{3}\vec{s}_{84})/2$, $\tilde{\vec{s}}_{35} = (\vec{s}_{35} + \sqrt{3}\vec{s}_{85})/2$, $\tilde{\vec{s}}_{36} = (-\vec{s}_{36} + \sqrt{3}\vec{s}_{86})/2$, $\tilde{\vec{s}}_{37} = (-\vec{s}_{37} + \sqrt{3}\vec{s}_{87})/2$, similar to $\tilde{L}_3$ and $\tilde{L}_8$.

To get (16) we used the identity: $\vec{s}_{ab}\vec{s}_{cd} - \vec{s}_{ad}\vec{s}_{cb} = \vec{s}_{ac}\vec{s}_{bd}$. The formula (16) possesses remarkable symmetry relative to the different $SU(2)$ subgroups of $SU(3)$. The functions L_8 or $\tilde{L}_8$ do not enter (16) as well as expression (6) for the baryon number density. The functions $\tilde{\omega}_i$ are connected with the body-fixed angular velocities of $SU(3)$ rotations by means of transformation (see (8) above)

$$\tilde{\omega} = V^\dagger \omega V - T\omega T^\dagger, \tag{17a}$$

or

$$\tilde{\omega}_i = (R_{ik}(V^\dagger) - R_{ik}(T))\omega_k = R_{ki}\omega_k. \tag{17b}$$

$R_{ik}(V^\dagger) = R_{ki}(V)$ and $R_{ik}(T)$ are real orthogonal matrices, $i,k = 1,...8$, and $\tilde{\omega}_i^2 = 2(\omega_i^2 - R_{kl}(U_0)\omega_k\omega_l)$. Expressions for R_{ik} are given in the Appendix for the general case of the parametrization (11). Relations (17) hold just because we are operating with rotated functions L_{ki}^r in (12).

The expression for static energy can be obtained from (16) by means of the substitution $\tilde{\omega}_i \to 2L_i$ and $\vec{s}_{ik} \to 2[\vec{L}_i\vec{L}_k]$, [16]. It can be written in a form which emphasizes quite clearly the lower boundary for the static energy proportional to the winding (baryon) number of the system:

$$E_{stat} = \int\Bigg\{\frac{F_\pi}{8e}\Bigg[(\vec{L}_1 - 2\vec{n}_{23} - \vec{n}_{47} + \vec{n}_{56})^2 + (\vec{L}_2 - 2\vec{n}_{31} - \vec{n}_{46} - \vec{n}_{57})^2 + (\vec{L}_4 - 2\vec{n}_{53} + \vec{n}_{17} + \vec{n}_{26})^2 +$$

$$+(\vec{L}_5 - 2\vec{n}_{34} - \vec{n}_{16} + \vec{n}_{27})^2 + (\vec{L}_6 - 2\vec{n}_{73} + \vec{n}_{15} - \vec{n}_{24})^2 + (\vec{L}_7 - 2\vec{n}_{36} - \vec{n}_{14} - \vec{n}_{25})^2 + \frac{2}{9}\Big([\vec{L}_3 + \tilde{\vec{L}}_3 - \frac{3}{2}(\vec{n}_{12} + \vec{n}_{45})]^2 +$$

$$+[\tilde{\vec{L}}_3 + \tilde{\tilde{\vec{L}}}_3 - \frac{3}{2}(\vec{n}_{45} + \vec{n}_{67})]^2 + [\tilde{\tilde{\vec{L}}}_3 - \vec{L}_3 - \frac{3}{2}(\vec{n}_{67} - \vec{n}_{12})]^2\Big)\Bigg] + M.t. + 3\pi^2\frac{F_\pi}{e}\tilde{B}\Bigg\}d^3\tilde{r}, \tag{18}$$

where $\tilde{B}$ is the baryon number density given by the integrand in (6), $\tilde{r} = F_\pi e r$ and $\vec{n}_{ik} = [\vec{L}_i\vec{L}_k]$. When $i = 4,5$, $k = 3$ then $\tilde{\vec{L}}_3$ should be taken in $\vec{n}_{i3}$. For $i = 6,7$ $\tilde{\tilde{L}}_3$ should be taken. In (18) we used relations $\tilde{L}_3 - \tilde{\tilde{L}}_3 = L_3$ and $(\vec{L}_3 + \tilde{\vec{L}}_3)^2 + (\tilde{\vec{L}}_3 + \tilde{\tilde{\vec{L}}}_3)^2 + (\tilde{\tilde{\vec{L}}}_3 - \vec{L}_3)^2 = \frac{9}{2}(\vec{L}_3^2 + \vec{L}_8^2)$. The chiral- and flavor-symmetry-breaking mass term $M.t.$ will be considered in Section 4.

From (18) we have the inequality

$$E_{stat} - M.t. \geq 3\pi^2\frac{F_\pi}{e}B. \tag{19}$$

This inequality was obtained first by Skyrme [1] for the $SU(2)$ model and is a particular case of the Bogomol'ny-type bound.

Eight diagonal moments of inertia and 28 off-diagonal ones define the rotation energy, a quadratic form in $\omega_i\omega_k$, according to (16), (17). The analytical expressions for the moments of inertia are too lengthy to be reproduced here. Fortunately, it is possible to perform calculations without explicit analytical formulas, by substituting (17) into (16).

The expression for E_{rot} simplifies considerably when the (u,d) $SU(2)$ soliton is quantized in the $SU(3)$ space of collective coordinates:

$$L_{rot}(SU_2) = \frac{F_\pi^2}{32}(\tilde{\omega}_1^2 + \tilde{\omega}_2^2 + ... + \tilde{\omega}_7^2) + \frac{1}{8e^2}\big(\vec{s}_{12}^{\,2} + \vec{s}_{23}^{\,2} + \vec{s}_{31}^{\,2} + \frac{1}{4}(\tilde{\omega}_4^2 + ... + \tilde{\omega}_7^2)(\vec{l}_1^2 + \vec{l}_2^2 + \vec{l}_3^2)\big), \tag{20a}$$

or

$$L_{rot}(SU_2) = \frac{F_\pi^2}{8}\big[\vec{\omega}^2\vec{f}^2 - (\vec{\omega}\vec{f})^2 + \frac{1-f_0}{2}(\omega_4^2 + \omega_5^2 + \omega_6^2 + \omega_7^2)\big] + \frac{1}{8e^2}\big[\vec{\omega}^2\vec{l}_i^{\,2} - (\vec{\omega}\vec{l}_i)^2 + \frac{1-f_0}{2}\vec{l}_i^2(\omega_4^2 + ... + \omega_7^2)\big], \tag{20b}$$

where $\tilde{\omega}_i = [R_{ik}(U_0) - \delta_{ik}]\omega_k = 2(f_i f_k - \vec{f}^2\delta_{ik} + f_0\epsilon_{ikl}f_l)\omega_k$ for $i, k = 1, 2, 3$, and $\vec{\omega}$ and $\vec{\tilde{\omega}}$ have three components in the (u, d) $SU(2)$ subgroup with $\vec{\tilde{\omega}}^2 = 4[\vec{\omega}^2\vec{f}^2 - (\vec{\omega}\vec{f})^2]$.

To derive $(20b)$ we used also that $\tilde{\omega}_4^2 + ... + \tilde{\omega}_7^2 = 2(1 - f_0)(\omega_4^2 + ... + \omega_7^2)$. Here $\vec{l}_i$ parametrizes the chiral derivatives of U_0: $U_0^\dagger d_k U_0 = i\tau_i l_{i,k}$, and the functions f_0, $\vec{f}$ define the matrix U_0 in the usual way. $\vec{l}_i^2 = (d_i f_0)^2 + ... + (d_i f_3)^2$.

Equation $(20b)$ defines the moments of inertia of arbitrary $SU(2)$ skyrmions rotated in $SU(3)$ configuration space and illustrates well that the $SU(2)$ case is much simpler than general $SU(3)$ case. The analytical expressions for the moments of inertia of axially symmetric $SU(2)$ skyrmions, also rotated in ν-direction, can be found in [11, 19].

When the $SU(2)$ hedgehog is quantized in the $SU(3)$ collective coordinates space only two different moments of inertia enter [8, 13, 14] : $\Theta_1 = \Theta_2 = \Theta_3$ and $\Theta_4 = \Theta_5 = \Theta_6 = \Theta_7$. For the $SO(3)$ hedgehog the rotation energy also depends on two different moments of inertia: $\Theta_2 = \Theta_5 = \Theta_7$ and $\Theta_1 = \Theta_3 = \Theta_4 = \Theta_6 = \Theta_8$ [8, 9]. In the case of the strange skyrmion molecule we obtained four different diagonal moments of inertia [20]: $\Theta_1 = \Theta_2 = \Theta_N$; Θ_3; $\Theta_4 = \Theta_5 = \Theta_6 = \Theta_7 = \Theta_S$ and Θ_8. Numerically the difference between Θ_N and Θ_3 is not large while Θ_8 is a bit greater than Θ_S (see Table 1). In view of the symmetry properties of the configuration many off-diagonal moments of inertia are equal to zero. Few of them are different from zero, but at least one order of magnitude smaller than the diagonal moments of inertia, e.g. Θ_{46} and Θ_{57}. For this reason we shall neglect them here in making estimates.

The Lagrangian of the system can be written in terms of the angular velocities of rotation and moments of inertia in the form (in the body-fixed system)

$$L_{rot} = \frac{\Theta_N}{2}(\omega_1^2 + \omega_2^2) + \frac{\Theta_3}{2}\omega_3^2 + \frac{\Theta_S}{2}(\omega_4^2 + \omega_5^2 + \omega_6^2 + \omega_7^2) + \frac{\Theta_8}{2}\omega_8^2 + \Theta_{45}(\omega_4\omega_5 - \omega_6\omega_7) + ... \tag{21}$$

The above-mentioned relations between the different moments of inertia of the strange molecule can be obtained in the following way, at large distances between the two $B = 1$ hedgehogs. When the $B = 1$ skyrmion is located in the (u, s) $SU(2)$ subgroup of $SU(3)$ we obtain from (12) and (16):

$$L_{rot}(u, s) = \frac{\theta_S}{2}(\omega_1^2 + \omega_2^2 + \omega_6^2 + \omega_7^2) + \frac{\theta_N}{2}[\omega_4^2 + \omega_5^2 + \frac{1}{4}(\omega_3 + \sqrt{3}\omega_8)^2], \tag{22a}$$

where we have retained the notations used for the (u, d) $B = 1$ soliton.

For the $B = 1$ skyrmion in the (d, s) subgroup:

$$L_{rot}(d, s) = \frac{\theta_S}{2}(\omega_1^2 + \omega_2^2 + \omega_4^2 + \omega_5^2) + \frac{\theta_N}{2}[\omega_6^2 + \omega_7^2 + \frac{1}{4}(\omega_3 - \sqrt{3}\omega_8)^2], \tag{22b}$$

with [13, 14]

$$\theta_S = \frac{1}{8}\int(1 - c_F)[F_\pi^2 + \frac{1}{e^2}(F'^2 + 2s_F^2/r^2)]d^3r$$
$$\theta_N = \frac{1}{6}\int s_F^2[F_\pi^2 + \frac{4}{e^2}(F'^2 + s_F^2/r^2)]d^3r, \tag{22c}$$

where $F(r)$ is the profile function of the $B = 1$ hedgehog and $f_0 = cosF$. Relations $(22c)$ follow immediately from $(20b)$. Note, that the combinations of ω_3 and ω_8 which enter the expressions for the rotation energy $(22a, b)$ and the WZW-term (10) are orthogonal to each other, as it follows from general arguments.

When two $B = 1$ hedgehogs in different subgroups, (d, s) and (u, s), are located at large distances, we should take the sum of the expressions $(22a)$, $(22b)$. Simple relations for the $B = 2$ moments of inertia Θ in terms of the $B = 1$ inertia θ then appear:

$$\Theta_N = 2\theta_S; \Theta_S = \theta_N + \theta_S; \Theta_3 = \theta_N/2; \Theta_8 = 3\theta_N/2 = 3\Theta_3. \tag{23}$$

For interacting hedgehogs in a molecule these relations hold only approximately (see Table 1 below where some numbers are corrected in comparison with [20]).

	B	M_{cl}	$M.t.$	C_S	Θ_N	Θ_S	Θ_3	Θ_8	Θ_{38}	$\Theta_{46} = -\Theta_{57}$
FS	1	1702	46	–	5.55	2.04	–	–	–	–
FS	2	3330	87	0.495	4.14	7.13	2.86	8.14	0.01	0.63
FSB	1	1982	199	–	3.24	1.06	–	–	–	–
FSB	2	3885	380	0.475	2.44	4.13	1.70	4.77	0.002	0.24

Table 1. The values of the masses M_{cl}, the mass term M.t. (in Mev), the strangeness content C_S and the moments of inertia (in $10^{-3}\ Mev^{-1}$) for the hedgehog with $B=1$ and the dipole configuration with $B=2$ [16] in the flavor-symmetric (FS) and flavor-symmetry-broken (FSB) cases. Here $M.t.$ is included in M_{cl}, $F_\pi = 186\ Mev$ and $e = 4.12$. The accuracy of calculations is at least $\sim 0.5\%$ in the masses and few % in other quantities.

In the flavor-symmetric (FS) case all meson masses in the Lagrangian are equal to the pion mass, the distance between centers of both skyrmions in the molecule equals ~ 1.05 Fm. In the FSB case the kaon mass is included in the Lagrangian (see the next section) and the distance between solitons centers in the molecule is $\sim 0.75\ Fm$ [16, 20].

The Hamiltonian of the system can be obtained by the canonical quantization procedure [2],[14],[8] which we reproduce here for completeness. The components of the body-fixed $SU(3)$ angular momentum J_k^R can be defined as

$$J_k^R = dL/d\omega_k \tag{24}$$

This definition coincides identically with another one,

$$J_k^R = \frac{1}{2i} Tr A\lambda_k \pi \tag{25}$$

where $\pi_{\alpha\beta} = dL/d\dot{A}_{\beta\alpha}$. In the canonical quantization procedure the substitution

$$\pi_{\alpha\beta} = \frac{dL}{d\dot{A}_{\beta\alpha}} \to -i\frac{d}{dA_{\beta\alpha}} \tag{26}$$

is made. The commutation relations

$$[J_i^R J_k^R] = -if_{ikl}J_l^R \tag{27}$$

then follow immediately, f_{ikl} are the $SU(3)$ structure constants.

After the standard quantization procedure the Hamiltonian of the system, $H = \omega_i dL/d\omega_i - L$, is a bilinear function of the generators J_i^R. For the states belonging to a definite $SU(3)$ irrep the rotation energy can be written in the simplified form:

$$E_{rot} = \frac{C_2(SU_3) - 3Y_R^2/4}{2\Theta_S} + \frac{N(N+1)}{2}\left(\frac{1}{\Theta_N} - \frac{1}{\Theta_S}\right) + \frac{3(Y_R - Y_R^{min})^2}{8\Theta_8} \tag{28}$$

The second order Casimir operator of the $SU(3)$ group is $C_2(SU_3) = \frac{1}{3}(p^2+q^2+pq)+p+q$, N is the right isospin (see Fig.2) and p, q are the numbers of the upper and low indices in the tensor describing the $SU(3)$ irrep (p,q). The terms linear in the angular velocities present in

the lagrangian due to the Wess-Zumino-Witten term are canceled in the Hamiltonian, but they lead to the quantization condition discussed in the previous section. Corrections of the order of Θ_{45}^2/Θ_S^2 and $(\Theta_N-\Theta_3)/\Theta_N$ have been neglected in (28). Note that Y_R^{min} can take arbitrary noninteger values because it is a quantity similar to the strangeness content C_S [17], not a quantum number. Y_R is a quantum number and can take only integer values. The usual spatial angular momentum is $J=0$ here. The correct description of the usual spatial rotations demands the introduction of a second set of collective coordinates, as it was done previously [11] for the case of flavor $SU(2)$. It was shown that the states of lowest energy have $J=0$.

It is clear from expression (28) that for $\Theta_8 \to 0$ the right hypercharge satisfies $Y_R = Y_R^{min} = \frac{2}{\sqrt{3}}L_8^{WZ}$, otherwise the quantum correction due to ω_8 will be infinite. For solitons located in (u,d) $SU(2)$ we have $\Theta_8 = 0$ and $Y_R = \frac{2}{\sqrt{3}}L_8^{WZ} = B$, the quantization condition [13, 18] with $N_c = 3$.

For the skyrmion molecule [16] we have $L_8^{WZ} \approx -\sqrt{3}/2$, or $Y_R^{min} \approx -1$, as was explained above. The last term in (25) is absent for $Y_R=-1$, and because of the evident constraints

$$\frac{p+2q}{3} \geq Y_R \geq -\frac{q+2p}{3} \tag{29}$$

the following lowest $SU(3)$ multiplets are possible: octet, $(p,q)=(1,1)$, decuplet $(3,0)$ and antidecuplet $(0,3)$, Fig.2. The sum of the classical mass of the soliton and rotational energy for the $B=2$ octet, 10 and $\bar{10}$ is equal to ~ 3.53, 3.74 and 3.89 Gev for $Y_R=-1$ (the flavor- symmetric FS-case). The whole FSB mass term described in the following section, $\Delta M + \delta M_{FS}$, should be added to these numbers. When the FSB mass term is included in the classical mass the sum $M_{cl} + E_{rot}$ equals 4.23, 4.59 and 4.84 Gev for the octet, decuplet and $\bar{10}$. Only the mass splitting part of the mass term, δM_{FSB}, should be added to these numbers (see Table 2 below). The octets with $Y_R = 0$ and 1 have $M_{cl}+E_{rot}+\Delta M$ equal to 4.61 and 4.73 Gev according to (28) (the FS scheme of calculation). The $SU(3)$ singlet with $Y_R=0$ has energy equal to $M_S = M_{cl} + 3/(8\Theta_8)$ which, according to Table 1 equals $\simeq 3.38$ Gev in the FS case. This can be compared with the $SO(3)$ hedgehog mass, $M_H = 3.272$ Gev for the same values of the parameters [12].

4 Mass splitting within $SU(3)$ multiplets of dibaryons

The mass splittings inside $SU(3)$ multiplets are defined as usual by the FSB part of the mass terms in the lagrangian density:

$$L_M = \frac{F_\pi^2 m_\pi^2}{16} Tr(U+U^\dagger-2) + \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{24} Tr(1-\sqrt{3}\lambda_8)(U+U^\dagger-2) \tag{30}$$

When (u,d) $SU(2)$ solitons are rotated in the "strange" direction by means of the matrix $U_4 = exp(-i\nu\lambda_4)$, (30) leads to the substitution $F_\pi^2 m_\pi^2 \to F_\pi^2 m_\pi^2 + sin^2\nu(F_K^2 m_K^2 - F_\pi^2 m_\pi^2)$ [4, 11]. For the ansatz (11), after averaging over all phases in the matrix $A(t)$ except ν, we can rewrite the mass term in the energy density in the following form:

$$M.t. = \frac{F_\pi^2 m_\pi^2}{8}(3-v_1-v_2-v_3) + \frac{F_K^2 m_K^2 - F_\pi^2 m_\pi^2}{4}[1 - v_3 + (2v_3 - v_1 - v_2)\frac{sin^2\nu}{2}], \tag{31a}$$

or, for $F_K = F_\pi$

$$M.t. = \frac{F_\pi^2 m_\pi^2}{4}\big[(3-v_1-v_2-v_3)\big(1/2 + (m_K^2/m_\pi^2 - 1)C_S\big) + (m_K^2/m_\pi^2-1)(2v_3-v_1-v_2)\frac{sin^2\nu}{2}.\big] \tag{31b}$$

Here v_1, v_2 and v_3 are real parts of the diagonal matrix elements of the matrix U, depending on the functions f_i and q_i. For the ansatz (11) we have ($b_0 = 0$)

$$v_1 = c_{a_0} c_a (c_b f_0 - s_b f_3) + s_{a_0} s_a c_b$$

$$v_2 = c_{a_0} c_a (c_b q_0 + s_b q_3) + s_{a_0} s_a [c_b (f_0 q_0 - f_3 q_3) + s_b (f_3 q_0 + f_0 q_3)] - s_{a_0} (f_1 q_1 + f_2 q_2)$$

$$v_3 = f_0 q_0 - f_3 q_3 + s_a [s_b (f_1 q_2 - f_2 q_1) - c_b (f_1 q_1 + f_2 q_2)] \qquad (32a)$$

a_0 and b_0 are the asymptotic values of the functions a, b. For the local minimum found recently [16] , $a_0 = b_0 = 0$. In this case $(32a)$ simplifies to

$$v_1 = c_a (c_b f_0 - s_b f_3),$$

$$v_2 = c_a (c_b q_0 + s_b q_3). \qquad (32b)$$

Here v_3 is given by $(32a)$ since it does not depend on a_0, b_0. If a_0, b_0 are different from zero the ansatz (11) should be written $U = exp(-i a_0 \lambda_2) U_L(u,s) U(u,d) U_R(d,s) exp(-i b_0 \lambda_3)$ to ensure the correct behaviour of $U(\vec{r})$ at $\vec{r} \to \infty$. For example, if $a = a_0 = \pi/2$, $b = b_0 = 0$ then $v_1 = 1$, $v_2 = v_3 = f_0 q_0 - f_3 q_3 - f_1 q_1 - f_2 q_2$, i.e., the skyrmion is located in the (d,s) $SU(2)$ subgroup.

In the FS case the part of the mass term

$$M.t._{FS} = F_\pi^2 m_\pi^2 (3 - v_1 - v_2 - v_3)/8 \qquad (33)$$

is included in the classical mass M_{cl} which is minimized. In the FSB case the second part,

$$\Delta M = (F_K^2 m_K^2 - F_\pi^2 m_\pi^2)(1 - v_3)/4 \qquad (34)$$

also is included in minimized M_{cl}, see Table 1. In the FS case $\Delta M \simeq 1016 Mev$, while in FSB case it is squeezed ~ 3 times.

The mass splitting inside $SU(3)$ multiplets is defined by the term

$$\delta M = -\frac{1}{4}(F_K^2 m_K^2 - F_\pi^2 m_\pi^2)(v_1 + v_2 - 2v_3) < \frac{1}{2} sin^2\nu >, \qquad (35)$$

which is not included in M_{cl} and is considered as a perturbation in both cases. Here ν is the angle of rotation in the "nonstrange" direction. For two undeformed hedgehogs at large relative distances we have $v_1 + v_2 - 2v_3 \to 2(1 - cosF)$ where F is the profile function of the $B = 1$ hedgehog, and the coefficient of $sin^2\nu$ is the same as for the rotated $B = 1$ (u,d) hedgehog. Note that in the case of a strange skyrmion molecule with strangeness content close to 0.5 the term (35) defining the mass splitting within multiplets is negative - directly opposite to the case when the nonstrange $SU(2)$ solitons are used as starting configurations and are rotated in the "strange" direction. The quantity δM should be added to the sum of $M_{cl} + E_{rot}$ calculated at the end of Section 3, and ΔM should be added in the FS case.

To obtain the mass splitting within $SU(3)$ multiplets we should calculate, as usual, the matrix elements of the function $< \frac{1}{2} sin^2\nu >= \frac{1}{3} < 1 - D_{88}(\nu) >= \frac{1}{3}(1 - I)$ for each component of the $SU(3)$ multiplets described by the $SU(3)$ $D-$functions. Then the quantity I is equal to

$$I = \sum_\gamma C^{8;(p.q);(p.q)_\gamma}_{0,0,0;Y,T,T_3;Y,T,T_3} C^{8;(p,q);(p,q)_\gamma}_{0,0,0;Y_R,N,M;Y_R,N,M}, \qquad (36)$$

expressed through the Clebsch-Gordan coefficients of the $SU(3)$ group [22]. In the case of a strange molecule we have $Y_R = -1$, $N = 1/2$ for the octet and decuplet, $N = 3/2$ for $\bar{10}$. The values of $< \frac{1}{2} sin^2\nu >$ and the mass splittings are shown in Table 2.

$\|p,q;Y,T>$	I	$-<sin^2\nu/2>$	δM_{FS}	M_{FS}	δM_{FSB}	M_{FSB}	$F.st.$	ϵ_{FS}
$\|8,1,1/2>$	$-2/10$	$-4/10$	-385	4.16	-124	4.10	ΛN	0.14
$\|8,0,1>$	$-1/10$	$-11/30$	-353	4.19	-114	4.11	ΞN	0.15
$\|8,0,0>$	$1/10$	$-3/10$	-289	4.26	-93	4.13	$\Lambda\Lambda$	0.14
$\|8,-1,1/2>$	$3/10$	$-7/30$	-224	4.32	-73	4.15	$\Lambda\Xi$	0.14
$\|10,1,3/2>$	$-1/8$	$-3/8$	-361	4.40	-117	4.47	ΣN	0.11
$\|10,0,1>$	0	$-1/3$	-320	4.44	-104	4.48	ΞN	0.11
$\|10,-1,1/2>$	$1/8$	$-7/24$	-280	4.48	-91	4.50	$\Lambda\Xi$	0.11
$\|10,-2,0>$	$1/4$	$-1/4$	-240	4.53	-78	4.52	$\Xi\Xi$	0.10
$\|\bar{1}0,2,0>$	$-1/4$	$-5/12$	-401	4.52	-130	4.72	NN	0.04
$\|\bar{1}0,1,1/2>$	$-1/8$	$-3/8$	-361	4.55	-117	4.72	ΛN	0.06
$\|\bar{1}0,0,1>$	0	$-1/3$	-320	4.59	-104	4.74	ΞN	0.07
$\|\bar{1}0,-1,3/2>$	$1/8$	$-7/24$	-280	4.63	-91	4.75	$\Sigma\Xi$	0.09

Table 2. The values of I, $\frac{1}{2}sin^2\nu$, the mass splitting δM (in Mev) and the masses M (in Gev) for the octet, decuplet and antidecuplet of dibaryons in the flavor-symmetric and flavor-symmetry-broken cases. The binding energy of the configuration $\epsilon = (M_1+M_2-M)/(M_1+M_2)$ relative to the final state $F.st.$ is presented. $M_{FS} = M_{cl,FS} + E_{rot,FS} + \Delta M + \delta M_{FS}$, $M_{FSB} = M_{cl,FSB} + E_{rot,FSB} + \delta M_{FSB}$.

For the octet the allowed strangeness of states is $-1, -2, -3$, for the decuplet it ranges from -1 to -4, and the nonstrange dibaryons appear in $\bar{1}0$, 27-plet, etc. (Fig.2). The masses of the dibaryons calculated according to the FS and FSB schemes differ, but not very much since the increase of the total mass term in the FS case is compensated by the decrease of E_{rot} in comparison with the FSB case. The states $|10,-2,0>$ and $|\bar{1}0,2,0>$ are supposed to have $J=1$ and the corresponding energy is added, roughly estimated according to our previous results [11].

When FSB mass terms are included in the minimized static energy M_{cl} they are squeezed by a factor ~ 3 due to the smaller dimensions of the kaon cloud in comparison with the pion cloud [16], therefore, the moments of inertia are greater and E_{rot} is smaller in the FS case (see Table 1). The absolute values of the masses are controlled by the Casimir energy [23]-[26], which has the order of magnitude $\sim -1Gev$ for $B=1$ [25, 26] and $\sim -2Gev$ for $B=2$ molecules.

For the 27-plet the value of the difference of I for states with maximum and minimum hypercharge is $3/8$, just as for decuplet and antidecuplet. The relative binding ϵ is shown in Table 2 because it is less sensitive to the method of calculation. M_1 and M_2 are the masses of the final baryons available due to strong interactions, calculated within the same approach (theory-to-theory comparison). Inclusion of configuration mixing usually leads to an increase of the mass splitting by $\sim 0.3-0.4$ [27]. Since the results for the mass splitting shown in Table 2 depend on the starting configuration, and both FS and FSB calculation schemes are not consistent by themselves, one should use some interpolating procedure, e.g., similar to the slow- rotator approximation used successfully in [4] for the description of the hyperon mass splitting.

5 Conclusions and discussion

The quantization scheme for the $SU(3)$ skyrmions has been presented and the quantization condition known previously [13] is generalized for skyrmions with arbitrary strangeness content, which allows one to investigate the consequences of the existence of different local

minima in $SU(3)$ configuration space. The quantization condition (8) is valid for all known $B = 2$ local minima shown in Fig.1. It is proved rigorously in several cases; in other cases it was confirmed by numerical calculation. However, some corrections to relation (8) cannot be excluded. The moments of inertia of arbitrary $SU(3)$ skyrmions can be calculated with the help of formulas $(16), (17)$. Both static and rotational energies as well as the baryon number density of $SU(3)$ skyrmions are presented in a form which makes apparent their symmetry in different $SU(2)$ subgroups of $SU(3)$.

For the dipole-type configuration with $C_S = 0.5$ our results are in qualitative agreement with those obtained in [28] for the interaction potential of two strange baryons located at large distances. The absolute values of the masses of both $B = 1$ and $B = 2$ states are controlled by the Casimir energies, which make a contribution of order N_c^0 to the masses of the configurations [22]-[25]. However, the dipole-type configuration does not differ much from the $B = 2$ configuration within the product ansatz which we used as a starting point in our calculations [16]. For this reason the Casimir energy of the dipole can be close to twice that for the $B = 1$ soliton and can cancel in the binding energies of dibaryons. We conclude therefore that a new branch of strange dibaryons in addition to those known previously [8, 9], [11] is predicted with a small uncertainty in the absolute values of masses due to the Casimir energy, relative to the corresponding $B = 1$ states. The values of masses and bindings we obtained here cannot, however, be taken too seriously, not only because the Casimir energy is poorly known but also because the non-zero mode contributions closely connected with the Casimir energy (principally the breathing and vibrational modes) have not been taken into account. These effects not only decrease the binding energies [6, 7], but can make many of the states listed in Table 2 unbound.

The prediction by chiral soliton models of a rich spectrum of baryonic states with different values of strangeness remains one of the intriguing properties of such models. The comparison with predictions of the quark or quark-bag models [29, 30] is of special interest. Some of such models predict the existence of bound strange baryonic states [30], similar to the chiral soliton approach.

It is difficult to observe these states, especially those which are above the threshold for decay due to strong interactions. The searches for the H-dibaryon predicted at first within the MIT quark-bag model [29] have been undertaken in different experiments, without success till now. It should be noted that observation of the H-dibaryon can be especially difficult by the following reasons. First, its dimensions are small in the framework of the chiral soliton approach [12, 11], $R_H \sim 0.5 - 0.6 Fm$. Therefore, estimates of the H-dibaryon production cross section based on the assumption that its dimensions are close to the dimensions of the deuteron may be too optimistic.

Second, it is not clear how the transition from an H-dibaryon to two $B = 1$ solitons can proceed. Schwesinger proposed a nontrivial parametrization allowing for the transition from the $SO(3)$ $B = 2$ hedgehog to the $B = 2$ $SU(2)$ torus (described in [31]). Within this parametrization the two configurations are separated by a potential barrier; moreover, the behaviour of some function in this parametrization is singular. So, if such a transition is not possible with smooth functions, it would be difficult to find H-dibaryon in coalescence experiments. However, further investigations of the predictions of effective field theories providing a new approach to the description of the fundamental properties of matter are of interest. The near-threshold enhancement in $p\Lambda$ system which was observed many years ago in, e.g., the reaction $pp \to p\Lambda K^+$ [32] and confirmed in recent investigations [33] may be a confirmation of soliton model predictions, because within this approach there is no

difference between real and virtual levels.

The problem of the H-dibaryon discussed in [8] is that of parity doubling: the $SO(3)$ soliton has no definite parity, so a special symmetrization procedure should be done [8]. A similar problem exists for the strange molecules also. For the classical configuration of molecular type we have different $B=1$ skyrmions in different parts of space and in different $SU(2)$ subgroups of $SU(3)$. The molecule has no definite parity, but these configurations are invariant under the combined operation of parity transformation and interchange of $SU(2)$ subgroups. The electric dipole momentum of the molecule is different from zero (this was noted by M.Luty). (Anti)symmetrization should be performed, similar to the H-particle case, providing a state of definite parity and removing the e.d.m. of the quantized state.

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6 Appendix

Here we sketch the expressions for the matrix elements R_{ik} which connect the rotation angular velocities in body-fixed and rotated coordinate systems: $\tilde{\omega}_i = R_{ki}\omega_k$, $R_{ik} = R_{ik}(V) - R_{ik}(T^+)$, $R_{ik}(T^+) = R_{ki}(T) = \frac{1}{2}Tr\lambda_i T^+ \lambda_k T$, $V = U_L(u,s)exp(ia\lambda_2)$, $T = exp(ib\lambda_3)U_R(d,s)$, $U_0 = VT$. Definitions of $U_L(u,s)$ and $U_R(d,s)$ in terms of functions $f_0,...f_3$ and $q_0,...q_3$ are given following expression (12).

We use the notations:

$$f_{12}^2 = f_1^2 + f_2^2,\ q_{12}^2 = q_1^2 + q_2^2,\ F_1^+ = f_0f_1 + f_2f_3,\ F_1^- = f_0f_1 - f_2f_3,$$
$$F_2^+ = f_0f_2 + f_1f_3,\ F_2^- = f_0f_2 - f_1f_3,\ F_3^+ = f_0f_3 + f_1f_2,\ F_3^- = f_0f_3 - f_1f_2,$$
$$Q_1^+ = q_0q_1 + q_2q_3,\ Q_1^- = q_0q_1 - q_2q_3,\ Q_2^+ = q_0q_2 + q_1q_3,\ Q_2^- = q_0q_2 - q_1q_3,$$
$$s_{bb_0} = sin(b-b_0),\ Q_c = c_bQ_2^- + s_bQ_1^+,\ Q_s = s_bQ_2^- - c_bQ_1^+,$$
$$\Delta_F = f_0^2 + f_1^2 - f_2^2 - f_3^2,\ \delta_F = f_0^2 - f_1^2 + f_2^2 - f_3^2,\ C^+ = c_{b+b_0}(q_1^2 - q_2^2) - 2s_{b+b_0}q_1q_2,$$
$$S^+ = s_{b+b_0}(q_1^2 - q_2^2) + 2c_{b+b_0}q_1q_2,\ C^- = c_{bb_0}(q_0^2 - q_3^2) + 2s_{bb_0}q_0q_3,\ S^- = s_{bb_0}(q_0^2 - q_3^2) - 2c_{bb_0}q_0q_3.\ (A1)$$

Here a_0 and b_0 are asymptotic values of functions a and b. For the strange molecule [16] we have $a_0 = b_0 = 0$. When, e.g., $a = a_0 = \pi/2$, $b = b_0 = 0$ hold, the matrix U corresponds to solitons located in the (d,s) $SU(2)$ subgroup of $SU(3)$.

$$R_{11} = s_{2a_0}s_{2a}(1 - f_{12}^2/2) + c_{2a_0}c_{2a}f_0 - c_{2b-2b_0}q_0 - s_{2b-2b_0}q_3$$
$$R_{12} = c_{2a_0}f_3 + s_{2b-2b_0}q_0 - c_{2b-2b_0}q_3,\ R_{13} = s_{2a_0}c_{2a}(1 - f_{12}^2/2) - c_{2a_0}s_{2a}f_0,$$
$$R_{14} = s_{2a_0}c_aF_2^+ - c_{2a_0}s_af_2 + c_{2b_0-b}q_2 + s_{2b_0-b}q_1,$$
$$R_{15} = -s_{2a_0}c_aF_1^- + c_{2a_0}s_af_1 - c_{2b_0-b}q_1 + s_{2b_0-b}q_2,$$
$$R_{16} = s_{2a_0}s_aF_2^+ + c_{2a_0}c_af_2,\ R_{17} = -s_{2a_0}s_aF_1^- - c_{2a_0}c_af_1,\ R_{18} = -\sqrt{3}s_{a_0}c_{a_0}f_{12}^2.$$
$$R_{21} = -c_{2a}f_3 - s_{2b-2b_0}q_0 + c_{2b-2b_0}q_3,\ R_{22} = f_0 - c_{2b-2b_0}q_0 - s_{2b-2b_0}q_3,$$
$$R_{23} = s_{2a}f_3,\ R_{24} = s_af_1 + c_{2b_0-b}q_1 - s_{2b_0-b}q_2,$$
$$R_{25} = s_af_2 + s_{2b_0-b}q_1 + c_{2b_0-b}q_2,\ R_{26} = -c_af_1,\ R_{27} = -c_af_2,\ R_{28} = 0.$$

$$R_{31} = s_{2a}c_{2a_0}(1 - f_{12}^2/2) - c_{2a}s_{2a_0}f_0,\ R_{32} = -s_{2a_0}f_3,$$
$$R_{33} = c_{2a}c_{2a_0}(1 - f_{12}^2/2) + s_{2a}s_{2a_0}f_0 - 1 + q_{12}^2/2,$$
$$R_{34} = c_ac_{2a_0}F_2^+ + s_as_{2a_0}f_2,\ R_{35} = -c_ac_{2a_0}F_1^- - s_as_{2a_0}f_1,$$

$R_{36} = s_a c_{2a_0} F_2^+ - c_a s_{2a_0} f_2 - Q_c,\ R_{37} = -s_a c_{2a_0} F_1^- + c_a s_{2a_0} f_1 - Q_s,$
$R_{38} = -\frac{\sqrt{3}}{2}(c_{2a_0} f_{12}^2 + q_{12}^2).$
$R_{41} = -s_{2a} c_{a_0} F_2^- + s_{a_0} c_{2a} f_2 - s_{2b-b_0} q_1 - c_{2b-b_0} q_2,$
$R_{42} = -s_{a_0} f_1 - c_{2b-b_0} q_1 + s_{2b-b_0} q_2,\ R_{43} = -c_{a_0} c_{2a} F_2^- - s_{a_0} s_{2a} f_2,$
$R_{44} = c_{a_0} c_a \Delta_F + s_{a_0} s_a f_0 - c_{bb_0} q_0 + s_{bb_0} q_3,$
$R_{45} = 2c_{a_0} c_a F_3^+ + s_{a_0} s_a f_3 + s_{bb_0} q_0 + c_{bb_0} f_3,$
$R_{46} = c_{a_0} s_a \Delta_F - s_{a_0} c_a f_0,\ R_{47} = 2c_{a_0} s_a F_3^+ - s_{a_0} c_a f_3,\ R_{48} = -\sqrt{3} c_{a_0} F_2^-.$

$R_{51} = c_{a_0} s_{2a} F_1^+ - s_{a_0} c_{2a} f_1 + c_{2b-b_0} q_1 - s_{2b-b_0} q_2,$
$R_{52} = -s_{a_0} f_2 - s_{2b-b_0} q_1 - c_{2b-b_0} q_2,\ R_{53} = c_{a_0} c_{2a} F_1^+ + s_{a_0} s_{2a} f_1,$
$R_{54} = -2c_{a_0} c_a F_3^- - s_{a_0} s_a f_3 - s_{bb_0} q_0 - c_{bb_0} q_3,$
$R_{55} = c_{a_0} c_a \delta_F + s_{a_0} s_a f_0 - c_{bb_0} q_0 + s_{bb_0} q_3,$
$R_{56} = -2c_{a_0} s_a F_3^- + s_{a_0} c_a f_3,\ R_{57} = c_{a_0} s_a \delta_F - s_{a_0} c_a f_0,\ R_{58} = \sqrt{3} c_{a_0} F_1^+.$
$R_{61} = -s_{a_0} s_{2a} F_2^- - c_{a_0} c_{2a} f_2,\ R_{62} = c_{a_0} f_1,$
$R_{63} = -s_{a_0} c_{2a} F_2^- + c_{a_0} s_{2a} f_2 + c_{b_0} Q_2^+ + s_{b_0} Q_1^-,$
$R_{64} = s_{a_0} c_a \Delta_F - c_{a_0} s_a f_0,\ R_{65} = 2s_{a_0} c_a F_3^+ - c_{a_0} s_a f_3,$
$R_{66} = s_{a_0} s_a \Delta_F + c_{a_0} c_a f_0 - C^- - C^+,\ R_{67} = 2s_{a_0} s_a F_3^+ + c_{a_0} c_a f_3 - S^- - S^+,$
$R_{68} = -\sqrt{3}[s_{a_0} F_2^- + c_{b_0} Q_2^+ + s_{b_0} Q_1^-].$

$R_{71} = s_{a_0} s_{2a} F_1^+ + c_{a_0} c_{2a} f_1,\ R_{72} = c_{a_0} f_2,$
$R_{73} = s_{a_0} c_{2a} F_1^+ - c_{a_0} s_{2a} f_1 - c_{b_0} Q_1^- + s_b Q_2^+,$
$R_{74} = -2s_{a_0} c_a F_3^- + c_{a_0} c_a f_3,\ R_{75} = s_{a_0} c_a \delta_F - c_{a_0} s_a f_0,$
$R_{76} = -2s_{a_0} s_a F_3^- - c_{a_0} c_a f_3 + S^- - S^+,\ R_{77} = s_{a_0} s_a \delta_F + c_{a_0} c_a f_0 - C^- + C^+,$
$R_{78} = \sqrt{3}[s_{a_0} F_1^+ - s_{b_0} Q_2^+ + c_{b_0} Q_1^-].$
$R_{81} = -\frac{\sqrt{3}}{2} s_{2a} f_{12}^2,\ R_{82} = 0,\ R_{83} = -\frac{\sqrt{3}}{2}(c_{2a} f_{12}^2 + q_{12}^2),\ R_{84} = \sqrt{3} c_a F_2^+,$
$R_{85} = -\sqrt{3} c_a F_1^-,\ R_{86} = \sqrt{3} s_a (F_2^+ + Q_C),\ R_{87} = -\sqrt{3} s_a (F_1^- - Q_S), R_{88} = \frac{3}{2}(q_{12}^2 - f_{12}^2)\ .\ (A2)$

The R_{8i} do not depend on a_0, b_0 because the matrices λ_2, λ_3 commute with λ_8. The orthogonality of the real matrices $R(V)$ and $R(T)$ can be checked immediately from these expressions.

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Figure captions

Fig.1 Map of the different local minima for classical configurations with $B = 2$ in the plane $(C_u - C_d)$, C_S. Here C_u, C_d and C_S are the scalar quark contents of the soliton, (1) is the $SO(3)$ hedgehog, (2),(3) and (4) are $SU(2)$ tori in the (u,d), (d,s) and (u,s) subgroups of $SU(3)$, and (5) is the dipole-type configuration (strange skyrmion molecule).

Fig.2 $T_3 - Y$ -diagrams for the lowest $SU(3)$ multiplets allowed for the case of $[SU(2)]^3$ configurations, ansatz (11): singlet $(p,q) = (0,0)$, octet $(1,1)$, decuplet $(3,0)$ and antidecuplet $(0,3)$. The lower dashed line indicates isomultiplets with $Y = -1 \simeq Y_R^{min}$, $T = N$; the upper dashed line shows nonstrange isomultiplets with $Y = B = 2$.